Published by Institute of Physics Publishing for SISSA/ISAS

Received: *August 6, 2004*
Revised: *November 8, 2004*
Accepted: *December 3, 2004*

AdSS$_5$ brane world cosmology

Denis Gonța

Department of Physics, St. Petersburg State University,
St. Petersburg, 198504, Russia
E-mail: d.gonta@pobox.spbu.ru

Abstract: In this paper, the gravitational equations of the 5-dimensional analogue of $AdSS$ space-time are investigated. Following methods of AdS/CFT correspondence and the black hole mechanics it is formulated the brane world model with the matter fields being confined on the 3-brane. The most general solutions are established in the generic case of an non-$\mathbb{Z}_2$-symmetric bulk and the energy-momentum tensor of the perfect fluid type. Constraining these solutions we derive a number of remarkable metrics widely investigated in the literature. Finally, we make many important conclusions about the viability of the presented scenario and cosmology.

Keywords: Extra Large Dimensions, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

Since the work of Kaluza and Klein [1] it has been an explicit possibility that there exist extra dimensions beyond those of Minkowski space-time. In recent years, the ideas of extra dimensions has become much more compelling. According to [2]–[4] it has been suggested that additional dimensions could have a quite distinct nature from our familiar dimensions, in other words, ordinary matter would be confined to our four-dimensional 'universe' while gravity would live in extended space-time. During the past 30 years, research in the theory of black holes in general relativity has brought to light strong hints of a fundamental relationship between gravitation, thermodynamics, and quantum theory. Indeed, the discovery of the thermodynamic behavior of black holes has given rise to most of our present physical insights into the nature of quantum phenomena occurring in strong gravitational fields. Recently several authors have found exact cosmological solutions in the brane world models, where a 3-brane is described as a domain wall moving in 5-dimensional black-hole like geometries [7]–[14] and [16]. Hence it is significant to examine how the bulk space-time is modified in the case of an non-$\mathbb{Z}_2$-symmetry and at the finite temperature.

In a typical brane world scenario [2] our 4-dimensional world $q_{\mu\nu}$ is described by a domain wall in 5-dimensional space-time g_{ab}[1]. The extra dimension need not be small or compact; in [3] Randall and Sundrum (RS) shown that gravity can be localized on a single brane even though the fifth dimension is noncompact. This noncompact localization arises via the introduction of a non-factorizable geometry. The metric contains a *warp factor* which is an exponential function of the extra dimension

$$ds^2 = e^{-2k|y|}(-dt^2 + d\vec{x}^{\,2}) + dy^2 \,. \tag{1.1}$$

The bulk is described by a AdS_5 metric, with $y = 0$ as a boundary, so that $y < 0$ is identified with $y > 0$ illustrating thus the $\mathbb{Z}_2$-symmetry.

The success of AdS/CFT correspondence [18]–[22] has lead to the intense study of these space-times. The AdS/CFT correspondence is a quite specific conjecture. For instance, this conjecture allows us to regard the entire AdS_5 geometry as a manifestation of the

[1]Where the Latin indices $a, b = 0, 1, 2, 3, 4$; and the Greek indices $\mu, \nu = 0, 1, 2, 3$.

dynamics of a 4-dimensional conformal field theory. It has been realized that there are deep connections between brane worlds scenario and AdS/CFT correspondence [20, 23, 24]. In this case, one should require two copies of the conformal field theory, one for each of the AdS_5 patches. One important point is that for a vacuum state of the theory, the extrinsic curvature of the boundary surface (brane) should be proportional to the induced metric [19, 21, 25]. For this reason, the bulk space-time is changed from AdS_5 to $AdSS_5$ with the metric

$$ds^2 = e^{-2y/\Lambda}\left[-\left(1-(\pi\Lambda T_0)^4 e^{4y/\Lambda}\right)dt^2 + d\vec{x}^{\,2}\right] + \frac{dy^2}{1-(\pi\Lambda T_0)^4 e^{4y/\Lambda}}\,, \tag{1.2}$$

where Λ (< 0) is the bulk cosmological constant and T_0 is the Hawking temperature realized through the periodic identification in the euclidean time (see [19, 6, 7] for discussions and references). This is a constant parameter of the $AdSS_5$ solution. We stress that the black-hole like geometries is an standard way to introduce the temperature [12, 13] and they are the main theoretical challenges today. At the same time taking into account the conventional brane worlds ideology one suitable candidate can be constructed by gluing together the two patches of $AdSS_5$ space-time along the brane world volume in a $\mathbb{Z}_2$-symmetric way [7]

$$ds^2 = e^{-2k|y|}\left[-\left(1-\frac{U_T^4}{k^4}\,e^{4k|y|}\right)dt^2 + d\vec{x}^{\,2}\right] + \frac{dy^2}{1-\frac{U_T^4}{k^4}\,e^{4k|y|}}\,, \tag{1.3}$$

where the horizon parameter U_T is proportional to the external temperature T_0.

Hence motivated by (1.2), (1.3) and other possible $AdSS_5$ like space-times we can assume without loss of generality the ansatz

$$ds^2 = e^{2\sigma(y)}\left[-\zeta(y)dt^2 + d\vec{x}^{\,2}\right] + \zeta(y)^{-1}dy^2\,. \tag{1.4}$$

In fact, this anzatz gives us a possibility to establish the most general solution of such family of space-times. This solution can be useful in order to clarify the symmetries and the main features of the $AdSS_5$ bulk and therefore to derive the induced Friedmann-Robertson-Walker (FRW) cosmology on the brane (see [19] for the radiation-dominated example). For the same reason, it is necessary to extend the scenario by adding matter to the brane.

2. Model building

Following methods of [8, 10] we shall suppose the location of the brane in the form $t = t(\tau)$, $y = y(\tau)$ parametrized by the local time τ on the brane. Then the induced metric on the brane can be expressed as follows

$$ds^2_{\rm ind} = -d\tau^2 + a(\tau)^2 d\vec{x}^{\,2} \equiv q_{\mu\nu}dx^\mu dx^\nu\,. \tag{2.1}$$

Obviously, the embedding of a brane is specified by the extrinsic curvature $K_{ab} = -(\delta_a^c - n_a n^c)\nabla_c n_b$ on both sides, with ∇_* being the covariant derivative compatible with (1.4). Suppose a single extrinsic curvature associated with the outward unit normal; then the proportionality condition between the extrinsic curvature and the induced metric takes the form [19]

$$K_{\mu\nu} = -\frac{1}{\Lambda}\, q_{\mu\nu}\,. \tag{2.2}$$

There is a set of solutions to (2.2) that relates the functions t and y in terms of $\sigma(y)$, $\zeta(y)$. In other words, we have only one function $y(\tau)$ that locates the brane and all the other surfaces have the same induced metric since they are given by translations in τ. Next step to proceed is to parametrize a particular surface by $(t_*, \vec{x})$, where t_* is a solution of (2.2) for t in terms of y. Then the induced metric becomes

$$ds^2_{\rm ind} = -e^{2\sigma(y)}\zeta(y)dt_*^2 + e^{2\sigma(y)}d\vec{x}^{\,2}\,, \tag{2.3}$$

which can be expressed in the standard FRW form (2.1) defining the $y = y(\tau)$ function appropriately. Notice that the authors of [8, 10] have introduced the relation between $t(\tau)$ and $y(\tau)$ in the form of a handy normalization condition. However in our case the relation is required by (2.2) in order to retain the vacuum state of the theory on the boundary [19].

For simplicity, we consider the confinement of the matter on a surface given by $\tau = \tau_*$ such that $y = y(\tau_*) = 0$, but keeping in mind that all the other surfaces can be obtained by translations in τ. Then the 5-dimensional Einstein equations are

$${}^{(5)}R_{ab} - \frac{1}{2}\, g_{ab}\, {}^{(5)}R = \widetilde{k}^2 T_{ab}\,; \qquad T_{ab} = -\Lambda g_{ab} + \sqrt{|\zeta(y)|}\, S_{ab}\delta(y)\,, \tag{2.4}$$

where $\widetilde{k}$ is the 5-dimensional gravitational coupling constant such that $\widetilde{k}^2 = 8\pi/{}^{(5)}M_p^3$, with ${}^{(5)}M_p$ the fundamental 5-dimensional Planck mass and

$$S_{ab} = (-\lambda q_{\mu\nu} + \tau_{\mu\nu})\delta^\mu_a \delta^\nu_b\,. \tag{2.5}$$

Notice that λ (> 0) and $\tau_{\mu\nu}$ are the intrinsic brane tension and the energy-momentum tensor of the brane that we require to be expressed in a more general perfect fluid form $\tau_{\mu\nu} = \mathrm{diag}\{-\widetilde{\rho}, \widetilde{p}_1, \widetilde{p}_2, \widetilde{p}_3\}$. It can easily be checked that (2.4) can be derived by taking the variation of the action

$$S = \int d^5x\sqrt{|g|}\left(\frac{{}^{(5)}M_p^3}{16\pi}\,{}^{(5)}R - \Lambda\right) + \int d^4x\sqrt{|q|}\left(\left[\frac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] - \lambda\right). \tag{2.6}$$

The energy density and the pressure given by $\tau_{\mu\nu}$ are expressed as follows

$$\widetilde{\rho} = -q_{00}\left(q^{00}\frac{1}{2}\,(\partial_0\varphi)^2 - V(\varphi)\right) \equiv q_{00}\rho\,, \qquad \widetilde{p}_\mu = -q_{\mu\mu}\left(q^{00}\frac{1}{2}\,(\partial_0\varphi)^2 + V(\varphi)\right) \equiv q_{\mu\mu}p\,.$$

Suppose that $p = \omega\rho$; then it is straightforward to show that the equations (2.4) yield three relations

$$\begin{aligned} \zeta(y)\sigma'(y)^2 + \frac{1}{4}\,\zeta'(y)\sigma'(y) &= k^2\,, \\ 2\zeta'(y)\sigma'(y) + \frac{1}{2}\,\zeta''(y) &= \widetilde{k}^2\sqrt{|\zeta(y)|}(1+\omega)\rho\,\delta(y)\,, \\ -3\sigma''(y)\sqrt{|\zeta(y)|} &= \widetilde{k}^2(\lambda+\rho)\delta(y)\,, \end{aligned} \tag{2.7}$$

where $k^2 \equiv -\widetilde{k}^2\Lambda/6$ and the prime denotes differentiation with respect to y. We stress that these relations are similar to the result for the first time obtained in [7] for $\rho = 0$.

JHEP12(2004)007

It is possible to derive the most general solution of (2.7) in a closed form

$$\sigma(y) = \alpha|y| + \beta y + \chi\,; \tag{2.8}$$

$$\zeta(y) = \gamma^2 + \left(\frac{\widetilde{k}^2\Lambda}{6(\alpha\epsilon(y)+\beta)^2} + \gamma^2\right)\left(e^{-4(\alpha|y|+\beta y)} - 1\right) \tag{2.9}$$

with the Israel junction conditions [5] at the brane

$$\frac{\beta^2}{\alpha^2} = \frac{12\Lambda}{\widetilde{k}^2[2(\lambda+\rho)^2 - 3(\lambda+\rho)(1+\omega)\rho]} + 1; \quad \gamma^2\alpha^2 = \frac{\widetilde{k}^4(\lambda+\rho)^2}{36}\,, \tag{2.10}$$

$$\text{where} \quad \lim_{y\to\pm 0}\zeta(y) \equiv \gamma^2; \quad \lim_{y\to\pm 0}\sigma(y) \equiv \chi \quad \text{and} \quad \epsilon(y) \equiv |y|\,' = \begin{cases} 1, & y>0 \\ -1, & y<0 \end{cases}\,.$$

It is seen that these solutions does not exhibit, in the general case, the $\mathbb{Z}_2$-symmetry and hence a number of restrictions should be imposed. To be precise, suppose that $\alpha = -k$, $\beta = 0$, $\chi = 0$, and $\rho = 0$; then (2.8)–(2.10) yield

$$\sigma(y) = -k|y|, \quad \zeta(y) = 1, \quad \lambda = \left(\frac{3\,{}^{(5)}\!M_p^3}{4\pi}\right)k\,. \tag{2.11}$$

This straightforwardly leads us to the warp factor and the intrinsic brane tension exactly as in the original RS scenario [2, 3]. It follows that in the $\mathbb{Z}_2$-symmetric case with no matter on the brane, we can achieve only an original RS like scenario. If one relax the assumption of $\mathbb{Z}_2$-symmetry ($\alpha = -k$, $\beta \neq 0$, $\chi = 0$, $\rho = 0$); then it can readily be checked that (2.8)–(2.10) yield

$$\zeta(y) = \frac{k^2}{(\beta - k\epsilon(y))^2} - \left(\frac{k^2}{(\beta - k\epsilon(y))^2} + \frac{k^2}{\beta^2 - k^2}\right)e^{4(k|y|-\beta y)}\,, \tag{2.12}$$

$$\sigma(y) = -k|y| + \beta y\,, \qquad \text{where} \qquad \beta^2 = k^2\left(1 - \frac{9\,{}^{(5)}\!M_p^6 k^2}{16\pi^2\lambda^2}\right). \tag{2.13}$$

Notice that the similar case was considered in the paper [7]. However the authors has focused on the equations like (2.7) being solved in the form of infinite series

$$\zeta(y) = 1 + \chi\zeta_1(y) + \chi^2\zeta_2(y) + \chi^3\zeta_3(y) + \chi^4\zeta_4(y) + \mathcal{O}(\chi^5)\,,$$
$$\sigma(y) = k|y| + \chi\sigma_1(y) + \chi^2\sigma_2(y) + \chi^3\sigma_3(y) + \chi^4\sigma_4(y) + \mathcal{O}(\chi^5)$$

with $\zeta_i(y)$, $\sigma_i(y)$ illustrating both the $\mathbb{Z}_2$-symmetric and $\mathbb{Z}_2$-antisymmetric behavior alternately. Apparently, summing these infinite series one should obtain the expressions of the type (2.12), (2.13). Besides, the inclusion of the matter gives us a possibility to consider the case $\alpha = -k$, $\beta = 0$, $\chi = 0$, and $\rho \neq 0$. The reader will have no difficulty in showing that the solutions (2.8), (2.9) can be reduced to the expression (1.3)

$$\sigma(y) = -k|y|\,, \qquad \zeta(y) = 1 - \frac{U_T^4}{k^4}\,e^{4k|y|}\,; \tag{2.14}$$

with two additional conditions in the form

$$\lambda = \frac{1}{4}\left(\rho(3\omega - 1) \pm \sqrt{9\rho^2(1+\omega)^2 - \frac{12\Lambda^{(5)}M_p^3}{\pi}}\right), \tag{2.15}$$

$$(1+\omega) = \frac{1}{\rho}\left(\frac{U_T^{4\,(5)}M_p^3}{2\pi k(k^4 + U_T^4)^{1/2}}\right) \equiv \frac{1}{\rho}\,\theta > 0\,. \tag{2.16}$$

Now recall that the pressure is related to the cosmological energy density via ω, this implies that $p = -\rho + \theta$. In fact, one can suppose that $\theta \ll \rho$; then $\rho \simeq -p$ and the matter sector has a suitable form for the slow-roll regime (see (2.6)).

Since we are interested in cosmological solutions, let us remember that we considered the confinement of the matter on a surface given by $\tau = \tau_*$ such that $y = y(\tau_*) = 0$. All the other surfaces have the same induced metric (2.1) related by translations in τ. The solutions (2.8), (2.9) and the junction conditions (2.10) are relevant in order to understand symmetries and the main features of the $AdSS_5$ bulk and therefore to derive the induced cosmology on the brane. Typical brane world scenarios assume a $\mathbb{Z}_2$ symmetry motivated by the M-Theory origin. Nevertheless many recent papers examine models that are not derived from M-Theory. For instance, there have been suggested multi-brane models and scenarios, where the bulk is not manifestly $\mathbb{Z}_2$-symmetric [8, 10, 11, 16] or one basically different action from (2.6) is proposed [17]. The conventional results that underline the non-$\mathbb{Z}_2$-symmetry reside in the effective Friedmann equations. Notice that the general brane embedding formalism have been constructed [15] and the extra terms beyond the standard $\mathbb{Z}_2$-symmetric case, which characterize the non-$\mathbb{Z}_2$-symmetric embedding have established. In our case, we do not derive such field equations. However the proposed brane model can be compared to the scenario [8, 10], where similar space-time is taken. Then the extra term must behaves like a positive Lambda-term for radiation or like a negative curvature term for dust (see also [11]). Further, I avoid presenting details since the calculations are not original to me. We stress that constraints on extra terms from nucleosynthesis must be taken into account as well [11, 17]. It can be expected that effects of the $\mathbb{Z}_2$-symmetry breaking terms are decreasing with the evolution of universe and hence at late times the standard brane world cosmology should be recovered [11, 16].

On the other hand, the bulk metrics (1.2) and (1.3) contain the terms that are proportional to the Hawking temperature. This can be understood as an external temperature that, in general, can be related to the boundary conformal field temperature via AdS/CFT correspondence [19]. In our case, the expression (2.16) suggests one possible candidate of such relation. However the finite temperature universe and the temperature induced on the brane are the topics in the course of development [12]–[14] and hence we leave the details for a forthcoming paper. Usually, the authors evaluate the energy (effective potential) of the quantum bulk matter fields on a $AdS_5(AdSS_5)$ background at nonzero temperature. The brane seems these thermal quantum effects via the effective potential that supplements the effective energy-momentum tensor. Finally, the existence of the thermal Casimir brane effect is also a subject of growing interest [12, 21] and should be taken into account.

JHEP12(2004)007

3. Conclusions

In this paper we have considered a brane cosmological scenario based on $AdSS_5$ spacetime. Taking account of AdS/CFT correspondence and the black-hole analogy we noticed a number of brane worlds candidates (1.2), (1.3) (see also [8]–[10] and [12]), which can be formulated in the $AdSS_5$ bulk and further, we introduced the anzatz (1.4). In the basic section we have obtained the most general solutions for this anzatz given by the action (2.6) and the condition (2.2). These solutions, in the general case, does not exhibit the $\mathbb{Z}_2$-symmetry and a number of restrictions should be imposed. We have proved that the solutions (2.8), (2.9) can lead us to the solutions of the original RS scenario [2, 3] or to the setup of the scenario proposed in [7].

The $AdSS_5$ brane cosmology is an widely investigated topic in the literature. However in our case, the AdS/CFT generic condition (2.2) imposes one physically relevant relation between the location functions $t(\tau)$ and $y(\tau)$. Hence it can be important to revise $AdSS_5$ based cosmologies taking account of the solutions (2.8), (2.9) and (2.2). Anyway, the arguments in this paper extend the results in the literature on the black-hole like geometries and the induced brane cosmologies.

Acknowledgments

The author is grateful to V.A. Franke, E.V. Prokhvatilov, and S.A. Paston for collaboration and helpful discussions. The author thanks also S.I. Vacaru for support.

References

[1] T. Kaluza, *Zum Unitätsproblem der Physik*, *Sitz. Preuss. Akad. Wiss. Phys. Math.* K **1** (1921) 966;
O. Klein, *The atomicity of electricity as a quantum theory law*, *Nature* **118** (1926) 516.

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[4] T. Shiromizu, K.I. Maeda and M. Sasaki, *The Einstein equations on the 3-brane world*, *Phys. Rev.* **D 62** (2000) 024012 [gr-qc/9910076].

[5] W. Israel, *Singular hypersurfaces and thin shells in general relativity*, *Nuovo Cim.* **B44** (1966) 4349.

[6] S.W. Hawking and D.N. Page, *Thermodynamics of black holes in anti-de Sitter space*, *Commun. Math. Phys.* **87** (1983) 577.

[7] D.K. Park, H. Kim, Y.G. Miao and H.J.W. Muller-Kirsten, *Randall-sundrum scenario at nonzero temperature*, *Phys. Lett.* **B 519** (2001) 159 [hep-th/0107156].

[8] P. Kraus, *Dynamics of anti-de Sitter domain walls*, *J. High Energy Phys.* **12** (1999) 011 [hep-th/9910149].

[9] P. Bowcock, C. Charmousis and R. Gregory, *General brane cosmologies and their global spacetime structure*, *Class. and Quant. Grav.* **17** (2000) 4745 [hep-th/0007177].

[10] D. Ida, *Brane-world cosmology*, *J. High Energy Phys.* **09** (2000) 014 [gr-qc/9912002].

[11] A.C. Davis, S.C. Davis, W.B. Perkins and I.R. Vernon, *Brane world phenomenology and the $\mathbb{Z}_2$ symmetry*, *Phys. Lett.* **B 504** (2001) 254 [hep-ph/0008132];
S.C. Davis, W.B. Perkins, A.-C. Davis and I.R. Vernon, *Cosmological phase transitions in a brane world*, *Phys. Rev.* **D 63** (2001) 083518 [hep-ph/0012223].

[12] S. Nojiri and S.D. Odintsov, *Can we live on the brane in schwarzschild-anti-de Sitter black hole?*, *Phys. Lett.* **B 493** (2000) 153 [hep-th/0007205];
S. Nojiri, S.D. Odintsov and S. Zerbini, *Bulk versus boundary (gravitational Casimir) effects in quantum creation of inflationary brane world universe*, *Class. and Quant. Grav.* **17** (2000) 4855 [hep-th/0006115].

[13] S. Shankaranarayanan, *Temperature and entropy of Schwarzschild-de Sitter space-time*, *Phys. Rev.* **D 67** (2003) 084026 [gr-qc/0301090].

[14] I. Brevik, K.A. Milton, S. Nojiri and S.D. Odintsov, *Quantum (in)stability of a brane-world AdS_5 universe at nonzero temperature*, *Nucl. Phys.* **B 599** (2001) 305 [hep-th/0010205].

[15] L.A. Gergely, *Generalized Friedmann branes*, *Phys. Rev.* **D 68** (2003) 124011 [gr-qc/0308072].

[16] N. Deruelle and T. Dolezel, *Brane versus shell cosmologies in Einstein and Einstein-Gauss-Bonnet theories*, *Phys. Rev.* **D 62** (2000) 103502 [gr-qc/0004021].

[17] R. Cordero and A. Vilenkin, *Stealth branes*, *Phys. Rev.* **D 65** (2002) 083519 [hep-th/0107175].

[18] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[19] S.S. Gubser, *AdS/CFT and gravity*, *Phys. Rev.* **D 63** (2001) 084017 [hep-th/9912001].

[20] M.J. Duff and J.T. Liu, *Complementarity of the Maldacena and Randall-Sundrum pictures*, *Phys. Rev. Lett.* **85** (2000) 2052 [hep-th/0003237].

[21] V. Balasubramanian and P. Kraus, *A stress tensor for anti-de Sitter gravity*, *Commun. Math. Phys.* **208** (1999) 413 [hep-th/9902121].

[22] H. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [hep-th/9906182].

[23] L. Anchordoqui, C. Núñez and K. Olsen, *Quantum cosmology and AdS/CFT*, *J. High Energy Phys.* **10** (2000) 050 [hep-th/0007064].

[24] S.W. Hawking, T. Hertog and H.S. Reall, *Brane new world*, *Phys. Rev.* **D 62** (2000) 043501 [hep-th/0003052].

[25] N.S. Deger and A. Kaya, *AdS/CFT and Randall-Sundrum model without a brane*, *J. High Energy Phys.* **05** (2001) 030 [hep-th/0010141].

JHEP12(2004)007